Noise and Distortion Figure – An Extension of Noise Figure Definition for Nonlinear Devices

Pedro Miguel Lavrador, Nuno Borges de Carvalho and José Carlos Pedro

Instituto de Telecomunicações, Universidade de Aveiro, 3810-193 Aveiro, Portugal.

***Abstract* — This paper deals in the integration of additive and nonlinear distortion noise contributions into a single new figure of merit: Noise and Distortion Figure, *NDF*. In the same way as traditional noise figure, *NF*, was conceived to be a measure of *SNR* degradation, *NDF* is now proposed as its extension to nonlinear systems, as a measure of *SINAD* degradation. *NDF* definition is discussed and its application to *SINAD* evaluations in systems of practical interest to the wireless community is exemplified.**

I. Introduction

The Noise Figure (*NF*), is an important figure of merit for designing low-noise systems. Its only drawback is that its validity is restricted to linear systems. Actually, real systems are not linear, and so their nonlinear distortion characteristics should also be incorporated in any high dynamic range design. The traditional approach consists in also taking, distortion figures as IP3, although in a separate way. Beyond their separate treatment of additive noise, these nonlinear distortion standards were measured using one or two tone test signals, and this kind of signals does not give a complete set of all possible nonlinear distortion effects. Although some work have already been done in this respect [1], it lacks generality as, again, only a single tone was used [2]

This paper proposes Noise and Distortion Figure, a new Figure of Merit that simultaneously handles additive noise and nonlinear distortion noise, identifying how these two perturbations will combine to affect the signal processed by the nonlinear system. The differences between Signal to Noise Ratio, *SNR*, and Signal to Noise and Distortion Ratio, *SINAD*, are pointed out first, and then an appropriate Noise and Distortion Figure of merit is defined accordingly. Finally, some simulations are made to exemplify its application and usefulness.

II. Signal to Noise Ratio versus Signal to Noise and Distortion Ratio.

It is usual, during the design of a RF system, to have it characterized in terms of noise via the Noise Figure, which gives a good measure of the addictive noise impact on a signal passed through that system. It is well known the straightforward relation between *NF* and Signal to Noise Ratio (*SNR*), defined as the ratio of signal power to noise power. *NF* is frequently referred has the ratio between input and output *SNRs*, although the formal definition of *NF* is:

$$NF = \frac{GN_o + N_a}{GN_o} \tag{1}$$

where, N_o and N_a are output available noise power spectral densities at a given source noise temperature, as seen if the system were noise free, and the system's added noise, respectively.

In a nonlinear system the approach described above is incomplete because the distortion noise produced by the nonlinearity (distortion components that have a stochastic behavior relative to the signal) is not taken in account. Another common figure of merit, which is more useful in the context of nonlinear systems is Signal to Noise and Distortion Ratio (*SINAD*) which is defined, according to [3], as the ratio of signal power density, to noise and distortion power densities, which can be written as:

$$SINAD_{lin}(\omega) = \frac{S(\omega)}{N(\omega) + D(\omega)} \tag{2}$$

where *S*, *N* and *D*, are respectively the Signal, additive Noise and Distortion power spectral densities. Since that the output distortion depends on the load, delivered power and not available power must be considered. Therefore hereinafter, power refers to power delivered to the load.

In order to evaluate and compare these two figures of merit consider a nonlinear system excited by an input, *x* composed of a signal *s* and noise *n*, the $SINAD_o$ can be calculated if we are able to evaluate the nonlinear output components. These components can be separated using a consequence of Price's Theorem [4], as was previously presented by Rowe [5], which stated that, for a memoryless nonlinearity $h(\cdot)$, with input *x*, the output $z=h(x)$, can be decomposed in

$$z(t) = \alpha \cdot x(t) + y(t) \tag{3}$$

Where $y(t)$ is uncorrelated with $x(t)$, and has two distinct components, the addictive noise introduced by the

0-7803-7695-1/03/$17.00 © 2003 IEEE

system and the produced nonlinear distortion. The origin of these two components is physically distinct so they are uncorrelated with each other and so they add in power. According to this formulation we have found our output signal component: $a.x(t)$, that is, we considered as signal every component that can be obtained by a gain factor from the input signal. The value a, is also calculated in [5], and can be interpreted as a cross-correlation of the output with the input signal. With all these statements we can write the $SINAD_o$.

$$SINAD_o(\omega)=\frac{\alpha^2\cdot S_i(\omega)}{\alpha^2\cdot N_i(\omega)+N_a+IMD(\omega)} \quad (4)$$

In this expression S_i, and N_i stand for the input signal and input noise power spectral densities respectively, a^2 is the equivalent linear power gain of the system, N_a the power density of addictive noise and IMD the power density of stochastic nonlinear Intermodulation Distortion.

III. – Noise and Distortion Figure.

It was already referred above that *NF* can represent the ratio of the input *SNR* (SNR_i) to the output *SNR* (SNR_o). If the same ratio is evaluated using *SINAD*, a figure identical to *NF* will be found except that it will now also include the distortion impact. Accordingly, we will call it Noise & Distortion Figure (*NDF*):

$$NDF(\omega)\equiv\frac{SINAD_i(\omega)}{SINAD_o(\omega)}=\frac{\dfrac{S_i(\omega)}{N_i(\omega)}}{\dfrac{\alpha^2\cdot S_i(\omega)}{\alpha^2\cdot N_i(\omega)+N_a+IMD(\omega)}}=\frac{\alpha^2\cdot N_i(\omega)+N_a+IMD(\omega)}{\alpha^2\cdot N_i(\omega)} \quad (5)$$

In (5) it can be seen that *NDF* does not depend only on the linear gain but also on the distortion produced. The main advantage of this new definition is that it allows simultaneous study of noise and distortion. This new figure of merit can be used without many effort compared with the usual *NF* designs.

Let us know obtain *NDF* for a typical third order polynomial nonlinearity, with Gaussian random inputs. It is already well known that the response of a polynomial nonlinearity similar to the one represented in Fig. 1 can be easily calculated [6].

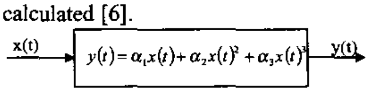

Fig. 1 The nonlinear model used.

The input signal considered in this case is composed of two independent Gaussian variables: one representing an arbitrary modulated signal, and another one representing the noise, as indicated in (6), where $s(t)$ and $n(t)$, stands for the signal and noise components respectively.

$$x(t)=s(t)+n(t) \quad (6)$$

Applying this signal to the nonlinearity represented in Fig. 1 will lead us two the output signal $y(t)$:

$$y(t)=\alpha_1(s(t)+n(t))+\alpha_2(s(t)+n(t))^2+\alpha_3(s(t)+n(t))^3 \quad (7)$$

Since when using random signals we must represent them with its autocorrelation function, we assume that, the input signal $x(t)$, has an autocorrelation function $R_{xx}(t) = R_{ss}(t)+R_{nn}(t)$. This is due to the uncorrelated behavior of n and s.

Using the result [6], which states that the output autocorrelation of a Gaussian signal passed through a third degree nonlinearity is given by:

$$R_{yy}(\tau)=\alpha_2^2R_{xx}(0)^2+\left[\alpha_1^2+6\alpha_1\alpha_3R_{xx}(0)+9\alpha_3^2R_{xx}(0)^2\right]\cdot R_{xx}(\tau)+2\alpha_2^2R_{xx}(\tau)^2+6\alpha_3^2R_{xx}(\tau)^3 \quad (8)$$

In the frequency domain:

$$\begin{aligned}S_{yy}(\omega)&=\alpha_2^2(R_{ss}(0)+R_{nn}(0))^2\delta(\omega)\\&+[\alpha_1+3\alpha_3(R_{ss}(0)+R_{nn}(0))]^2\cdot(S_{ss}(\omega)+S_{nn}(\omega))\\&+2\alpha_2^2[S_{ss}(\omega)*S_{ss}(\omega)+S_{nn}(\omega)*S_{nn}(\omega)+2S_{nn}(\omega)*S_{ss}(\omega)]\\&+6\alpha_3^2[S_{ss}(\omega)*S_{ss}(\omega)*S_{ss}(\omega)+S_{nn}(\omega)*S_{nn}(\omega)*S_{nn}(\omega)\\&\quad+3\cdot S_{nn}(\omega)*S_{nn}(\omega)*S_{ss}(\omega)+3\cdot S_{nn}(\omega)*S_{ss}(\omega)*S_{ss}(\omega)]\end{aligned} \quad (9)$$

In this expression it can be seen that there is a component of the input signal and noise that emerges at the output simply affected by a gain factor which depends on a_1, a_3 and on the total input power. Note that the second order components appear clearly out of band: one component is DC and the other component is at the second harmonic zone. The extra perturbation to the signal is produced by the third order component which combines to appear at the fundamental output zone and at the third harmonic zone.

For a particular case of the input, where the signals are flat over a bandwidth B, with power P_s and P_n.

$$S_{ss}(\omega)=\begin{cases}P_s/2B, & -\omega_H\le\omega\le-\omega_L,\omega_L\le\omega\le\omega_H\\0 & ,elsewhere\end{cases}$$

$$S_{nn}(\omega)=\begin{cases}P_n/2B, & -\omega_H\le\omega\le-\omega_L,\omega_L\le\omega\le\omega_H\\0 & ,elsewhere\end{cases} \quad (10)$$

$$SINAD_o(\omega) = \frac{[\alpha_1 + 3\alpha_3(P_s + P_n)]^2 \, P_s/2B}{[\alpha_1 + 3\alpha_3(P_s + P_n)]^2 \, P_n/2B + 6\alpha_3^2\left(-\omega^2 + (\omega_L + \omega_H)\omega + \frac{B^2}{2} - \omega_L\omega_H\right) \cdot 3\cdot (P_s + P_n)^3/8B^3 + N_a} \tag{12}$$

$$NDF(\omega) = \frac{[\alpha_1 + 3\alpha_3(P_s + P_n)]^2 P_n + \frac{9}{2}\alpha_3^2\left(-\omega^2 + (\omega_L + \omega_H)\omega + \frac{B^2}{2} - \omega_L\omega_H\right)(P_s + P_n)^3/B^2 + N_a}{[\alpha_1 + 3\alpha_3(P_s + P_n)]^2 P_n} \tag{13}$$

The output power spectral density in the fundamental zone may be written as:

$$\begin{aligned} S_{yy}(\omega) &= \left[\alpha_1^2 + 6\alpha_1\alpha_3(P_s + P_n) + 9\alpha_3^2(P_s + P_n)^2\right]\cdot(P_s + P_n)/2B \\ &+ 6\alpha_3^2\left(-\omega^2 + (\omega_L + \omega_H)\omega + \frac{B^2}{2} - \omega_L\omega_H\right) \\ &\cdot \frac{3}{8B^3}\left(P_s^3 + P_n^3 + 3P_s^2P_n + 3P_sP_n^2\right) \end{aligned} \tag{11}$$

Separating the signal and non-signal components in (11), and taking the effect of additive noise, the output SINAD in band can be obtained as depicted in expression (12).

Using, expression (5) and (12), *NDF* in band for this case is presented as expression (13), where we can see that the *NDF* assumes a parabolic pattern inside the band. That is due to the triple convolution of the bandpass signal used in this example.

IV. PRACTICAL EXAMPLES

In order to exemplify some of these theoretical results, the NDF of a simple system, as the one of Figure 2 was simulated, and the results compared with the theoretical formulae developed above.

On Figure 2, to obtain the input for the Non Linear Function, NLF, a bandpass Gaussian signal is generated, and added to bandpass noise in order to obtain the given SNR_i. The power of this sum, signal and noise, is then adjusted in order to test the NLF in different zones. The noise contribution is then added to the NLF's output, and the cross-correlation of the output and input calculated to find the NLF equivalent linear gain. The linear output signal and noise due to the input is calculated via a linear amplifier, and these two components subtracted from the output to find the nonlinear distortion and additive noise.

The power densities of the three components are measured in the mid band, and $SINAD_o$ calculated.

Simulation results of *NDF* of three interesting variants of the system are shown in Fig. 3, along with the theoretically predicted *NDF* by expression (13). These correspond to *NDF* dependence on total input power for 1 - Linear system; 2 - Nonlinear system with gain compression (polynomial coefficients [1 0 -0.01]) and 3 - Nonlinear system presenting a gain expansion (polynomial coefficients [1 0 0.01]). The noise power density used was $N_a = 10^{-4}\,\mathrm{WHz}^{-1}$. The $SINAD_i$ 30 dB, and input power (signal plus noise) as indicated in the figures.

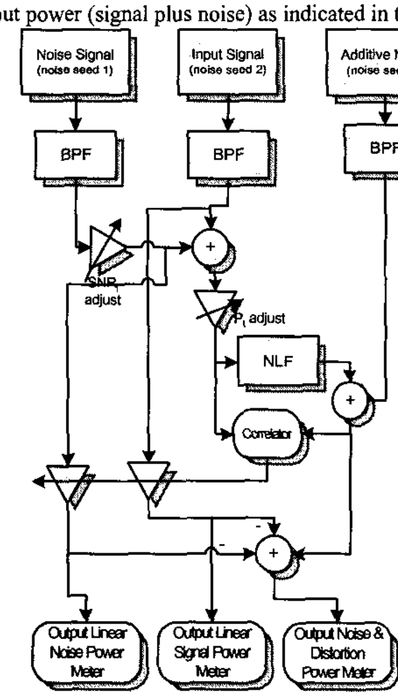

Fig. 2. Block Diagram of the simulator used to evaluate NDF.

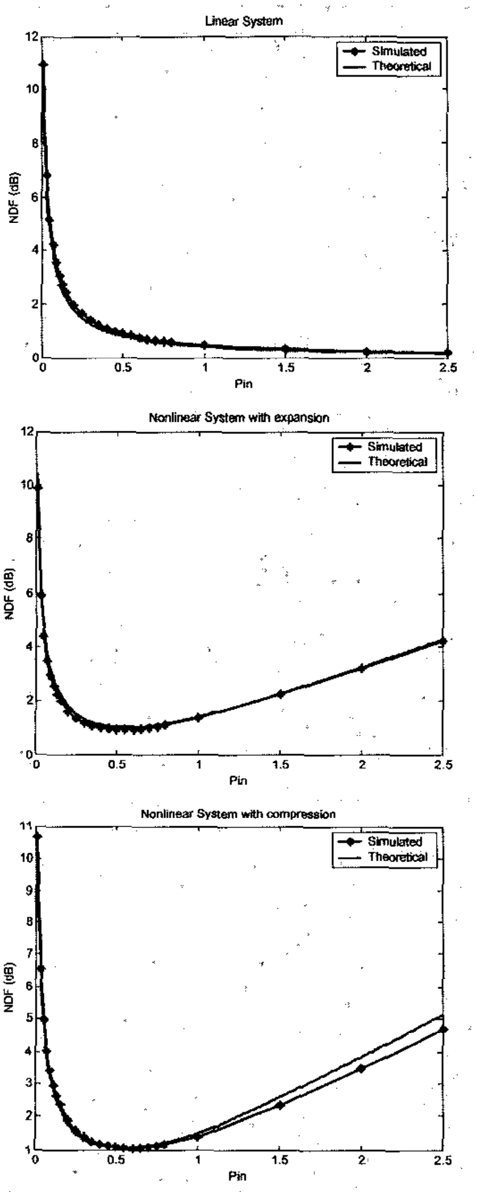

Fig. 3 Comparison of expression 12 and simulation results. a)Linear System. b) Nonlinear system with gain expansion expansion. c) Nonlinear system with gain compression.

Fig. 3 shows a good agreement between the simulations made and the theoretical predictions. It should be noted that in Fig. 3-a), the linear case, the curve for *NDF* is identical to the usual *NF* curve as was expected from the definition of (5).

V. CONCLUSIONS

A new figure of merit was proposed to integrate noise and distortion impact on the degradation of signal quality. Besides being defined for the general case, it was then calculated for the particular example of a memoryless non linearity. Good results have been found when comparing the predictions given by the theoretical expression with the simulation results.

ACKNOWLEDGEMENT

The authors would like to acknowledge the financial support provided by Portuguese science bureau, F.C.T., under Project POCTI/ESE/37531/2002 – OPAMS.

The first author also thanks the Ph.D. grant of F.C.T. ref 6835/2001, under which the work was done.

REFERENCES

[1] A. Geens and Y. Rolain "Noise Figure Measurements on Nonlinear Devices", *IEEE Trans. on Instrumentation and Meas.*, vol. IM-50 no. 4 , Aug. 2001.

[2] Westcott, R. J., "Investigation of Multiple FM/FDM Carriers Through a Satellite TWT Operating Near to Saturation", *Proceedings of the IEE*, vol. 114, no.6, pp.726-740, 1967.

[3] T. C. Hofner, "Defining and testing dynamic ADC parameters", Microwaves & RF, November 2000

[4] R. Price, "A Useful Theorem for nonlinear devices having Gaussian inputs", *IEEE Trans. Inform. Theory*, IT -4, pp. 69-72, June 1958.

[5] H. E. Rowe, "Memoryless nonlinearities with Gaussian inputs: elementary results", *The Bell System Tech. Journal*, vol. 71, no.7, pp 1519-1525, Sep. 1982.

[6] J. C. Pedro and N. B. Carvalho, "On the Use of Multitone Techniques for Assessing RF Components' Intermodulation Distortion", *IEEE Trans. on Microwave Theory and Tech.*, Volume: 47 Issue: 12 , Dec. 1999